Genericity of Representations of $p$-Adic $Sp_{2n}$ and Local Langlands Parameters

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Abstract. Let $G$ be the $F$-rational points of the symplectic group $Sp_{2n}$, where $F$ is a non-Archimedean local field of characteristic 0. Cogdell, Kim, Piatetski-Shapiro, and Shahidi constructed local Langlands functorial lifting from irreducible generic representations of $G$ to irreducible representations of $GL_{2n+1}(F)$. Jiang and Soudry constructed the descent map from irreducible supercuspidal representations of $GL_{2n+1}(F)$ to those of $G$, showing that the local Langlands functorial lifting from the irreducible supercuspidal generic representations is surjective. In this paper, based on above results, using the same descent method of studying $SO_{2n+1}$ as Jiang and Soudry, we will show the rest of local Langlands functorial lifting is also surjective, and for any local Langlands parameter $\phi \in \Phi(G)$, we construct a representation $\sigma$ such that $\phi$ and $\sigma$ have the same twisted local factors. As one application, we prove the $G$-case of a conjecture of Gross-Prasad and Rallis, that is, a local Langlands parameter $\phi \in \Phi(G)$ is generic, i.e., the representation attached to $\phi$ is generic, if and only if the adjoint $L$-function of $\phi$ is holomorphic at $s = 1$. As another application, we prove for each Arthur parameter $\psi$, and the corresponding local Langlands parameter $\phi_\psi$, the representation attached to $\phi_\psi$ is generic if and only if $\phi_\psi$ is tempered.

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