Generating Functions for Hecke Algebra Characters

Matjaž Konvalinka and Mark Skandera

Abstract. Certain polynomials in $n^2$ variables that serve as generating functions for symmetric group characters are sometimes called $(S_n)$ character immanants. We point out a close connection between the identities of Littlewood–Merris–Watkins and Goulden–Jackson, which relate $S_n$ character immanants to the determinant, the permanent and MacMahon’s Master Theorem. From these results we obtain a generalization of Muir’s identity. Working with the quantum polynomial ring and the Hecke algebra $H_n(q)$, we define quantum immanants that are generating functions for Hecke algebra characters. We then prove quantum analogs of the Littlewood–Merris–Watkins identities and selected Goulden–Jackson identities that relate $H_n(q)$ character immanants to the quantum determinant, quantum permanent, and quantum Master Theorem of Garoufalidis–Lê–Zeilberger. We also obtain a generalization of Zhang’s quantization of Muir’s identity.

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.
e-mail: matjaz@mit.edu

Department of Mathematics, Lehigh University, Bethlehem, PA 18015, U.S.A.
e-mail: mas906@lehigh.edu

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