Multiple Zeta-Functions Associated with Linear Recurrence Sequences and the Vectorial Sum Formula

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Abstract. We prove the holomorphic continuation of certain multi-variable multiple zeta-functions whose coefficients satisfy a suitable recurrence condition. In fact, we introduce more general vectorial zeta-functions and prove their holomorphic continuation. Moreover, we show a vectorial sum formula among those vectorial zeta-functions from which some generalizations of the classical sum formula can be deduced.

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