Abstract. Let $\mu$ be a nonnegative Radon measure on $\mathbb{R}^d$ that satisfies the growth condition that there exist constants $C_0 > 0$ and $n \in (0, d]$ such that for all $x \in \mathbb{R}^d$ and $r > 0$, $\mu(B(x, r)) \leq C_0 r^n$, where $B(x, r)$ is the open ball centered at $x$ and having radius $r$. In this paper, the authors prove that if $f$ belongs to the BMO-type space $\text{RBMO}(\mu)$ of Tolsa, then the homogeneous maximal function $\dot{M}_S(f)$ (when $\mathbb{R}^d$ is not an initial cube) and the inhomogeneous maximal function $M_S(f)$ (when $\mathbb{R}^d$ is an initial cube) associated with a given approximation of the identity $S$ of Tolsa are either infinite everywhere or finite almost everywhere, and in the latter case, $\dot{N}_S$ and $N_S$ are bounded from $\text{RBMO}(\mu)$ to the BLO-type space $\text{RBLO}(\mu)$. The authors also prove that the inhomogeneous maximal operator $N_S$ is bounded from the local BMO-type space $\text{rbmo}(\mu)$ to the local BLO-type space $\text{rblo}(\mu)$. 

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