Characterizations of Extremals for some Functionals on Convex Bodies

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Abstract. We investigate equality cases in inequalities for Sylvester-type functionals. Namely, it was proven by Campi, Colesanti, and Gronchi that the quantity
\[ \int_{x_0 \in K} \cdots \int_{x_n \in K} [V(\text{conv}\{x_0, \ldots, x_n\})]^p \, dx_0 \cdots dx_n, \quad n \geq d, \quad p \geq 1 \]
is maximized by triangles among all planar convex bodies \( K \) (parallelograms in the symmetric case). We show that these are the only maximizers, a fact proven by Giannopoulos for \( p = 1 \). Moreover, if \( h: \mathbb{R}_+ \to \mathbb{R}_+ \) is a strictly increasing function and \( W_j \) is the \( j \)-th quermassintegral in \( \mathbb{R}^d \), we prove that the functional
\[ \int_{x_0 \in K_0} \cdots \int_{x_n \in K_n} h(W_j(\text{conv}\{x_0, \ldots, x_n\})) \, dx_0 \cdots dx_n, \quad n \geq d \]
is minimized among the \((n+1)\)-tuples of convex bodies of fixed volumes if and only if \( K_0, \ldots, K_n \) are homothetic ellipsoids when \( j = 0 \) (extending a result of Groemer) and Euclidean balls with the same center when \( j > 0 \) (extending a result of Hartzoulaki and Paouris).

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