Revisiting Tietze–Nakajima: Local and Global Convexity for Maps

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Abstract. A theorem of Tietze and Nakajima, from 1928, asserts that if a subset $X$ of $\mathbb{R}^n$ is closed, connected, and locally convex, then it is convex. We give an analogous “local to global convexity” theorem when the inclusion map of $X$ to $\mathbb{R}^n$ is replaced by a map from a topological space $X$ to $\mathbb{R}^n$ that satisfies certain local properties. Our motivation comes from the Condevaux–Dazord–Molino proof of the Atiyah–Guillemin–Sternberg convexity theorem in symplectic geometry.

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Received by the editors February 26, 2008.
Published electronically July 6, 2010.
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The first author is partially supported by an NSERC Discovery grant. The second author was partially funded by an NSERC USRA grant in the summers of 2004 and 2005.
AMS subject classification: 53D20, 52B99.