The Brascamp–Lieb Polyhedron

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Abstract. A set of necessary and sufficient conditions for the Brascamp–Lieb inequality to hold has recently been found by Bennett, Carbery, Christ, and Tao. We present an analysis of these conditions. This analysis allows us to give a concise description of the set where the inequality holds in the case where each of the linear maps involved has co-rank 1. This complements the result of Barthe concerning the case where the linear maps all have rank 1. Pushing our analysis further, we describe the case where the maps have either rank 1 or rank 2.

A separate but related problem is to give a list of the finite number of conditions necessary and sufficient for the Brascamp–Lieb inequality to hold. We present an algorithm which generates such a list.

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