Some Applications of the Perturbation Determinant in Finite von Neumann Algebras

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Abstract. In the finite von Neumann algebra setting, we introduce the concept of a perturbation determinant associated with a pair of self-adjoint elements $H_0$ and $H$ in the algebra and relate it to the concept of the de la Harpe–Skandalis homotopy invariant determinant associated with piecewise $C^1$-paths of operators joining $H_0$ and $H$. We obtain an analog of Krein’s formula that relates the perturbation determinant and the spectral shift function and, based on this relation, we derive subsequently (i) the Birman–Solomyak formula for a general non-linear perturbation, (ii) a universality of a spectral averaging, and (iii) a generalization of the Dixmier–Fuglede–Kadison differentiation formula.

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