Tail Bounds for the Stable Marriage of Poisson and Lebesgue

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Abstract. Let $\Xi$ be a discrete set in $\mathbb{R}^d$. Call the elements of $\Xi$ centers. The well-known Voronoi tessellation partitions $\mathbb{R}^d$ into polyhedral regions (of varying volumes) by allocating each site of $\mathbb{R}^d$ to the closest center. Here we study allocations of $\mathbb{R}^d$ to $\Xi$ in which each center attempts to claim a region of equal volume $\alpha$.

We focus on the case where $\Xi$ arises from a Poisson process of unit intensity. In an earlier paper by the authors it was proved that there is a unique allocation which is stable in the sense of the Gale–Shapley marriage problem. We study the distance $X$ from a typical site to its allocated center in the stable allocation.

The model exhibits a phase transition in the appetite $\alpha$. In the critical case $\alpha = 1$ we prove a power law upper bound on $X$ in dimension $d = 1$. (Power law lower bounds were proved earlier for all $d$). In the non-critical cases $\alpha < 1$ and $\alpha > 1$ we prove exponential upper bounds on $X$. 

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