Close Lattice Points on Circles

Javier Cilleruelo and Andrew Granville

Abstract. We classify the sets of four lattice points that all lie on a short arc of a circle that has its center at the origin; specifically on arcs of length $t R^{1/3}$ on a circle of radius $R$, for any given $t > 0$. In particular we prove that any arc of length $(40 + \frac{40}{3} \sqrt{10})^{1/3} R^{1/3}$ on a circle of radius $R$, with $R > \sqrt{65}$, contains at most three lattice points, whereas we give an explicit infinite family of 4-tuples of lattice points, $(\nu_1, n, \nu_2, n, \nu_3, n, \nu_4, n)$, each of which lies on an arc of length $(40 + \frac{40}{3} \sqrt{10})^{1/3} R^{1/3} + o(1)$ on a circle of radius $R_n$.

Instituto de Ciencias Matemáticas (CSIC-UAM-UC3M-UCM) and Departamento de Matemáticas, Universidad Autónoma de Madrid, 28049 Madrid, Spain
E-mail: franciscojavier.cilleruelo@uam.es

Département de Mathématiques et Statistique, Université de Montréal, CP 6128 succ Centre-Ville, Montréal, QC H3C 3J7
E-mail: andrew@dms.umontreal.ca

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