The Face Semigroup Algebra of a Hyperplane Arrangement

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Abstract. This article presents a study of an algebra spanned by the faces of a hyperplane arrangement. The quiver with relations of the algebra is computed and the algebra is shown to be a Koszul algebra. It is shown that the algebra depends only on the intersection lattice of the hyperplane arrangement. A complete system of primitive orthogonal idempotents for the algebra is constructed and other algebraic structure is determined including: a description of the projective indecomposable modules, the Cartan invariants, projective resolutions of the simple modules, the Hochschild homology and cohomology, and the Koszul dual algebra. A new cohomology construction on posets is introduced, and it is shown that the face semigroup algebra is isomorphic to the cohomology algebra when this construction is applied to the intersection lattice of the hyperplane arrangement.

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Received by the editors August 30, 2006.
AMS subject classification: Primary: 52C35; secondary: 05E25, 16S37.
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