Generalized Polynomials and Mild Mixing

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Abstract. An unsettled conjecture of V. Bergelson and I. Håland proposes that if \((X, \mathcal{A}, \mu, T)\) is an invertible weak mixing measure preserving system, where \(\mu(X) < \infty\), and if \(p_1, p_2, \ldots, p_k\) are generalized polynomials (functions built out of regular polynomials via iterated use of the greatest integer or floor function) having the property that no \(p_i\), nor any \(p_i - p_j, i \neq j\), is constant on a set of positive density, then for any measurable sets \(A_0, A_1, \ldots, A_k\), there exists a zero-density set \(E \subset \mathbb{Z}\) such that

\[
\lim_{n \to \infty} \mu(A_0 \cap T^{p_1(n)}A_1 \cap \cdots \cap T^{p_k(n)}A_k) = \prod_{i=0}^{k} \mu(A_i).
\]

We formulate and prove a faithful version of this conjecture for mildly mixing systems and partially characterize, in the degree two case, the set of families \(\{p_1, p_2, \ldots, p_k\}\) satisfying the hypotheses of this theorem.

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