Hauteur asymptotique des points de Heegner

En l’honneur du Professeur Henryk Iwaniec, pour l’ensemble de son oeuvre analytique et pour son titre de Docteur Honoris Causa de l’Université de Bordeaux 1.

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Abstract. Geometric intuition suggests that the Néron–Tate height of Heegner points on a rational elliptic curve $E$ should be asymptotically governed by the degree of its modular parametrisation. In this paper, we show that this geometric intuition asymptotically holds on average over a subset of discriminants. We also study the asymptotic behaviour of traces of Heegner points on average over a subset of discriminants and find a difference according to the rank of the elliptic curve. By the Gross–Zagier formulae, such heights are related to the special value at the critical point for either the derivative of the Rankin–Selberg convolution of $E$ with a certain weight one theta series attached to the principal ideal class of an imaginary quadratic field or the twisted $L$-function of $E$ by a quadratic Dirichlet character. Asymptotic formulae for the first moments associated with these $L$-series and $L$-functions are proved, and experimental results are discussed. The appendix contains some conjectural applications of our results to the problem of the discretisation of odd quadratic twists of elliptic curves.