A Characterization of the Quantum Cohomology Ring of $G/B$ and Applications

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Abstract. We observe that the small quantum product of the generalized flag manifold $G/B$ is a product operation $\ast$ on $H^*_c(G/B) \otimes \mathbb{R}[q_1, \ldots, q_l]$ uniquely determined by the facts that it is a deformation of the cup product on $H^*_c(G/B)$; it is commutative, associative, and graded with respect to $\deg(q_i) = 4$; it satisfies a certain relation (of degree two); and the corresponding Dubrovin connection is flat. Previously, we proved that these properties alone imply the presentation of the ring $(H^*_c(G/B) \otimes \mathbb{R}[q_1, \ldots, q_l], \ast)$ in terms of generators and relations. In this paper we use the above observations to give conceptually new proofs of other fundamental results of the quantum Schubert calculus for $G/B$: the quantum Chevalley formula of D. Peterson (see also Fulton and Woodward) and the "quantization by standard monomials" formula of Fomin, Gelfand, and Postnikov for $G = SL(n, \mathbb{C})$. The main idea of the proofs is the same as in Amarzaya–Guest: from the quantum $D$-module of $G/B$ one can decode all information about the quantum cohomology of this space.