Genus 2 Curves with Quaternionic Multiplication
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Abstract. We explicitly construct the canonical rational models of Shimura curves, both analytically in terms of modular forms and algebraically in terms of coefficients of genus 2 curves, in the cases of quaternion algebras of discriminant 6 and 10. This emulates the classical construction in the elliptic curve case. We also give families of genus 2 QM curves, whose Jacobians are the corresponding abelian surfaces on the Shimura curve, and with coefficients that are modular forms of weight 12. We apply these results to show that our \( j \)-functions are supported exactly at those primes where the genus 2 curve does not admit potentially good reduction, and construct fields where this potentially good reduction is attained. Finally, using \( j \), we construct the fields of moduli and definition for some moduli problems associated to the Atkin–Lehner group actions.