Furstenberg Transformations and Approximate Conjugacy

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Abstract. Let $\alpha$ and $\beta$ be two Furstenberg transformations on 2-torus associated with irrational numbers $\theta_1$, $\theta_2$, integers $d_1$, $d_2$ and Lipschitz functions $f_1$ and $f_2$. It is shown that $\alpha$ and $\beta$ are approximately conjugate in a measure theoretical sense if (and only if) $\theta_1 \pm \theta_2 = 0$ in $\mathbb{R}/\mathbb{Z}$. Closely related to the classification of simple amenable $C^*$-algebras, it is shown that $\alpha$ and $\beta$ are approximately $K$-conjugate if (and only if) $\theta_1 \pm \theta_2 = 0$ in $\mathbb{R}/\mathbb{Z}$ and $|d_1| = |d_2|$. This is also shown to be equivalent to the condition that the associated crossed product $C^*$-algebras are isomorphic.