

On Effective Witt Decomposition and the Cartan–Dieudonné Theorem

Lenny Fukshansky

Abstract. Let K be a number field, and let F be a symmetric bilinear form in $2N$ variables over K . Let Z be a subspace of K^N . A classical theorem of Witt states that the bilinear space (Z, F) can be decomposed into an orthogonal sum of hyperbolic planes and singular and anisotropic components. We prove the existence of such a decomposition of small height, where all bounds on height are explicit in terms of heights of F and Z . We also prove a special version of Siegel’s lemma for a bilinear space, which provides a small-height orthogonal decomposition into one-dimensional subspaces. Finally, we prove an effective version of the Cartan–Dieudonné theorem. Namely, we show that every isometry σ of a regular bilinear space (Z, F) can be represented as a product of reflections of bounded heights with an explicit bound on heights in terms of heights of F , Z , and σ .

Received by the editors January 24, 2005.

AMS subject classification: Primary: 11E12, 15A63; secondary: 11G50.

Keywords: quadratic forms, heights.

©Canadian Mathematical Society 2007.