Calderón–Zygmund Operators Associated to Ultraspherical Expansions

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Abstract. We define the higher order Riesz transforms and the Littlewood–Paley $g$-function associated to the differential operator $L_{\lambda} f(\theta) = -f'''(\theta) - 2\lambda \cot \theta f''(\theta) + \lambda^2 f'(\theta)$. We prove that these operators are Calderón–Zygmund operators in the homogeneous type space $(0, \pi, (\sin t)^{\lambda} dt)$. Consequently, $L^p$ weighted, $H^1 - L^1$ and $L^\infty - \text{BMO}$ inequalities are obtained.