

## Searching for Absolute $\mathcal{CR}$ -Epic Spaces

*We dedicate this paper to the memory of John Isbell, 1930-2005*

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*Abstract.* In previous papers, Barr and Raphael investigated the situation of a topological space  $Y$  and a subspace  $X$  such that the induced map  $C(Y) \rightarrow C(X)$  is an epimorphism in the category  $\mathcal{CR}$  of commutative rings (with units). We call such an embedding a  $\mathcal{CR}$ -epic embedding and we say that  $X$  is absolute  $\mathcal{CR}$ -epic if every embedding of  $X$  is  $\mathcal{CR}$ -epic. We continue this investigation. Our most notable result shows that a Lindelöf space  $X$  is absolute  $\mathcal{CR}$ -epic if a countable intersection of  $\beta X$ -neighbourhoods of  $X$  is a  $\beta X$ -neighbourhood of  $X$ . This condition is stable under countable sums, the formation of closed subspaces, cozero-subspaces, and being the domain or codomain of a perfect map. A strengthening of the Lindelöf property leads to a new class with the same closure properties that is also closed under finite products. Moreover, all  $\sigma$ -compact spaces and all Lindelöf  $P$ -spaces satisfy this stronger condition. We get some results in the non-Lindelöf case that are sufficient to show that the Dieudonné plank and some closely related spaces are absolute  $\mathcal{CR}$ -epic.

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