Weighted Inequalities for Hardy–Steklov Operators

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Abstract. We characterize the pairs of weights \((v, w)\) for which the operator \(T f(x) = g(x) \int_{s(x)}^{h(x)} f\) with \(s\) and \(h\) increasing and continuous functions is of strong type \((p, q)\) or weak type \((p, q)\) with respect to the pair \((v, w)\) in the case \(0 < q < p\) and \(1 < p < \infty\). The result for the weak type is new while the characterizations for the strong type improve the ones given by H. P. Heinig and G. Sinnamon. In particular, we do not assume differentiability properties on \(s\) and \(h\) and we obtain that the strong type inequality \((p, q), q < p\), is characterized by the fact that the function

\[ \Phi(x) = \sup \left( \frac{\int_c^d g^q w}{\left( \int_c^d v^{1/p} \right)^{1/p}} \right) \]

belongs to \(L^{1/r}(g^q w)\), where \(1/r = 1/q - 1/p\) and the supremum is taken over all \(c\) and \(d\) such that \(c \leq x \leq d\) and \(s(d) \leq h(c)\).