Smooth Values of the Iterates of the Euler Phi-Function

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Abstract. Let \( \phi(n) \) be the Euler phi-function, define \( \phi_0(n) = n \) and \( \phi_{k+1}(n) = \phi(\phi_k(n)) \) for all \( k \geq 0 \). We will determine an asymptotic formula for the set of integers \( n \) less than \( x \) for which \( \phi_k(n) \) is \( y \)-smooth, conditionally on a weak form of the Elliott–Halberstam conjecture.