On Fiber Cones of $m$-Primary Ideals

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Abstract. Two formulas for the multiplicity of the fiber cone $F(I) = \bigoplus_{n=0}^{\infty} I^n/mI^n$ of an $m$-primary ideal of a $d$-dimensional Cohen–Macaulay local ring $(R, m)$ are derived in terms of the mixed multiplicity $e_{d-1}(m|I)$, the multiplicity $e(I)$, and superficial elements. As a consequence, the Cohen–Macaulay property of $F(I)$ when $I$ has minimal mixed multiplicity or almost minimal mixed multiplicity is characterized in terms of the reduction number of $I$ and lengths of certain ideals. We also characterize the Cohen–Macaulay and Gorenstein properties of fiber cones of $m$-primary ideals with a $d$-generated minimal reduction $J$ satisfying $\ell(I^2/JI) = 1$ or $\ell(I^m/Jm) = 1$.