Classification of Ding’s Schubert Varieties: Finer Rook Equivalence

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Abstract. K. Ding studied a class of Schubert varieties $X_\lambda$ in type A partial flag manifolds, indexed by integer partitions $\lambda$ and in bijection with dominant permutations. He observed that the Schubert cell structure of $X_\lambda$ is indexed by maximal rook placements on the Ferrers board $B_\lambda$, and that the integral cohomology groups $H^*(X_\lambda; \mathbb{Z}), H^*(X_\mu; \mathbb{Z})$ are additively isomorphic exactly when the Ferrers boards $B_\lambda, B_\mu$ satisfy the combinatorial condition of rook-equivalence.

We classify the varieties $X_\lambda$ up to isomorphism, distinguishing them by their graded cohomology rings with integer coefficients. The crux of our approach is studying the nilpotence orders of linear forms in the cohomology ring.