

Partial $*$ -Automorphisms, Normalizers, and Submodules in Monotone Complete C^* -Algebras

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Abstract. For monotone complete C^* -algebras $A \subset B$ with A contained in B as a monotone closed C^* -subalgebra, the relation $X = AsA$ gives a bijection between the set of all monotone closed linear subspaces X of B such that $AX + XA \subset X$ and $XX^* + X^*X \subset A$ and a set of certain partial isometries s in the “normalizer” of A in B , and similarly for the map $s \mapsto \text{Ad } s$ between the latter set and a set of certain “partial $*$ -automorphisms” of A . We introduce natural inverse semigroup structures in the set of such X 's and the set of partial $*$ -automorphisms of A , modulo a certain relation, so that the composition of these maps induces an inverse semigroup homomorphism between them. For a large enough B the homomorphism becomes surjective and all the partial $*$ -automorphisms of A are realized via partial isometries in B . In particular, the inverse semigroup associated with a type II_1 von Neumann factor, modulo the outer automorphism group, can be viewed as the fundamental group of the factor. We also consider the C^* -algebra version of these results.

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