Partial $\ast$-Automorphisms, Normalizers, and Submodules in Monotone Complete $C^\ast$-Algebras

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Abstract. For monotone complete $C^\ast$-algebras $A \subseteq B$ with $A$ contained in $B$ as a monotone closed $C^\ast$-subalgebra, the relation $X = AxA$ gives a bijection between the set of all monotone closed linear subspaces $X$ of $B$ such that $AX +XA \subseteq X$ and $XX^* + X^*X \subseteq A$ and a set of certain partial isometries $s$ in the “normalizer” of $A$ in $B$, and similarly for the map $s \mapsto Ad_s$ between the latter set and a set of certain “partial $\ast$-automorphisms” of $A$. We introduce natural inverse semigroup structures in the set of such $X$’s and the set of partial $\ast$-automorphisms of $A$, modulo a certain relation, so that the composition of these maps induces an inverse semigroup homomorphism between them. For a large enough $B$ the homomorphism becomes surjective and all the partial $\ast$-automorphisms of $A$ are realized via partial isometries in $B$. In particular, the inverse semigroup associated with a type II$_1$ von Neumann factor, modulo the outer automorphism group, can be viewed as the fundamental group of the factor. We also consider the $C^\ast$-algebra version of these results.

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