Diametrically Maximal and Constant Width Sets in Banach Spaces

Dedicated to the memory of Simon Fitzpatrick

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Abstract. We characterize diametrically maximal and constant width sets in $C(K)$, where $K$ is any compact Hausdorff space. These results are applied to prove that the sum of two diametrically maximal sets needs not be diametrically maximal, thus solving a question raised in a paper by Groemer. A characterization of diametrically maximal sets in $\ell^1_2$ is also given, providing a negative answer to Groemer’s problem in finite dimensional spaces. We characterize constant width sets in $c_0(I)$, for every $I$, and then we establish the connections between the Jung constant of a Banach space and the existence of constant width sets with empty interior. Porosity properties of families of sets of constant width and rotundity properties of diametrically maximal sets are also investigated. Finally, we present some results concerning non-reflexive and Hilbert spaces.

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