Stark’s Conjecture and New Stickelberger Phenomena

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Abstract. We introduce a new conjecture concerning the construction of elements in the annihilator ideal associated to a Galois action on the higher-dimensional algebraic $K$-groups of rings of integers in number fields. Our conjecture is motivic in the sense that it involves the (transcendental) Borel regulator as well as being related to $l$-adic étale cohomology. In addition, the conjecture generalises the well-known Coates–Sinnott conjecture. For example, for a totally real extension when $r = -2, -4, -6, \ldots$ the Coates–Sinnott conjecture merely predicts that zero annihilates $K_{-2}$ of the ring of $S$-integers while our conjecture predicts a non-trivial annihilator. By way of supporting evidence, we prove the corresponding (conjecturally equivalent) conjecture for the Galois action on the étale cohomology of the cyclotomic extensions of the rationals.