Multiplicty Results for Nonlinear Neumann Problems

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Abstract. In this paper we study nonlinear elliptic problems of Neumann type driven by the $p$-Laplacian differential operator. We look for situations guaranteeing the existence of multiple solutions. First we study problems which are strongly resonant at infinity at the first (zero) eigenvalue. We prove five multiplicity results, four for problems with nonsmooth potential and one for problems with a $C^1$-potential. In the last part, for nonsmooth problems in which the potential eventually exhibits a strict super-$p$-growth under a symmetry condition, we prove the existence of infinitely many pairs of nontrivial solutions. Our approach is variational based on the critical point theory for nonsmooth functionals. Also we present some results concerning the first two elements of the spectrum of the negative $p$-Laplacian with Neumann boundary condition.