Asymptotic Behavior of the Length of Local Cohomology

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Abstract. Let $k$ be a field of characteristic 0, $R = k[x_1, \ldots, x_d]$ be a polynomial ring, and $m$ its maximal homogeneous ideal. Let $I \subset R$ be a homogeneous ideal in $R$. Let $\lambda(M)$ denote the length of an $R$-module $M$. In this paper, we show that

$$\lim_{n \to \infty} \frac{\lambda(H^0_m(R/\mathfrak{p}^n))}{n^d} = \lim_{n \to \infty} \frac{\lambda(\text{Ext}^d_R(R/\mathfrak{p}^n, R(-d)))}{n^d}$$

always exists. This limit has been shown to be $e(I)/d!$ for $m$-primary ideals $I$ in a local Cohen–Macaulay ring, where $e(I)$ denotes the multiplicity of $I$. But we find that this limit may not be rational in general. We give an example for which the limit is an irrational number thereby showing that the lengths of these extension modules may not have polynomial growth.