The Gelfond–Schnirelman Method in Prime Number Theory

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Abstract. The original Gelfond–Schnirelman method, proposed in 1936, uses polynomials with integer coefficients and small norms on \([0, 1]\) to give a Chebyshev-type lower bound in prime number theory. We study a generalization of this method for polynomials in many variables. Our main result is a lower bound for the integral of Chebyshev’s \(\psi\)-function, expressed in terms of the weighted capacity. This extends previous work of Nair and Chudnovsky, and connects the subject to the potential theory with external fields generated by polynomial-type weights. We also solve the corresponding potential theoretic problem, by finding the extremal measure and its support.