Generalized $k$-Configurations

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Abstract. In this paper, we find configurations of points in $n$-dimensional projective space ($\mathbb{P}^n$) which simultaneously generalize both $k$-configurations and reduced 0-dimensional complete intersections. Recall that $k$-configurations in $\mathbb{P}^2$ are disjoint unions of distinct points on lines and in $\mathbb{P}^n$ are inductively disjoint unions of $k$-configurations on hyperplanes, subject to certain conditions. Furthermore, the Hilbert function of a $k$-configuration is determined from those of the smaller $k$-configurations. We call our generalized constructions $k_D$-configurations, where $D = \{d_1, \ldots, d_r\}$ (a set of $r$ positive integers with repetition allowed) is the type of a given complete intersection in $\mathbb{P}^n$. We show that the Hilbert function of any $k_D$-configuration can be obtained from those of smaller $k_D$-configurations. We then provide applications of this result in two different directions, both of which are motivated by corresponding results about $k$-configurations.