Smoothness of Quotients Associated With a Pair of Commuting Involutions

Aloysius G. Helminck and Gerald W. Schwarz

Abstract. Let $\sigma$, $\theta$ be commuting involutions of the connected semisimple algebraic group $G$ where $\sigma$, $\theta$ and $G$ are defined over an algebraically closed field $k$, $\text{char} \ k = 0$. Let $H := G^\sigma$ and $K := G^\theta$ be the fixed point groups. We have an action $(H \times K) \times G \rightarrow G$, where $((h, k), g) \mapsto hgk^{-1}$, $h \in H$, $k \in K$, $g \in G$. Let $G/(H \times K)$ denote the categorical quotient $\text{Spec} \ O(G)^{H \times K}$. We determine when this quotient is smooth. Our results are a generalization of those of Steinberg [Ste75], Pittie [Pit72] and Richardson [Ric82] in the symmetric case where $\sigma = \theta$ and $H = K$.