Type Decomposition and the Rectangular AFD Property for $W^*$-TRO’s

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Abstract. We study the type decomposition and the rectangular AFD property for $W^*$-TRO’s. Like von Neumann algebras, every $W^*$-TRO can be uniquely decomposed into the direct sum of $W^*$-TRO’s of type $I$, type $II$, and type $III$. We may further consider $W^*$-TRO’s of type $I_{m,n}$ with cardinal numbers $m$ and $n$, and consider $W^*$-TRO’s of type $II_{\lambda,\mu}$ with $\lambda, \mu = 1$ or $\infty$. It is shown that every separable stable $W^*$-TRO (which includes type $I_{\infty,\infty}$, type $II_{\infty,\infty}$ and type $III$) is TRO-isomorphic to a von Neumann algebra. We also introduce the rectangular version of the approximately finite dimensional property for $W^*$-TRO’s. One of our major results is to show that a separable $W^*$-TRO is injective if and only if it is rectangularly approximately finite dimensional. As a consequence of this result, we show that a dual operator space is injective if and only if its operator predual is a rigid rectangular $OL_{1,1}$-space (equivalently, a rectangular $OL_{1,1}$-space).

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