Asymptotics for Minimal Discrete Riesz Energy on Curves in $\mathbb{R}^d$

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Abstract. We consider the $s$-energy $E(Z_n; s) = \sum_{i \neq j} K(\|z_{i,n} - z_{j,n}\|; s)$ for point sets $Z_n = \{z_{k,n} : k = 0, \ldots, n\}$ on certain compact sets $\Gamma$ in $\mathbb{R}^d$ having finite one-dimensional Hausdorff measure, where

$$K(t ; s) = \begin{cases} t^{-s}, & \text{if } s > 0, \\ -\ln t, & \text{if } s = 0, \end{cases}$$

is the Riesz kernel. Asymptotics for the minimum $s$-energy and the distribution of minimizing sequences of points is studied. In particular, we prove that, for $s \geq 1$, the minimizing nodes for a rectifiable Jordan curve $\Gamma$ distribute asymptotically uniformly with respect to arclength as $n \to \infty$. 

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