The Geometry of Quadratic Differential Systems with a Weak Focus of Third Order

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Abstract. In this article we determine the global geometry of the planar quadratic differential systems with a weak focus of third order. This class plays a significant role in the context of Hilbert’s 16-th problem. Indeed, all examples of quadratic differential systems with at least four limit cycles, were obtained by perturbing a system in this family. We use the algebro-geometric concepts of divisor and zero-cycle to encode global properties of the systems and to give structure to this class. We give a theorem of topological classification of such systems in terms of integer-valued affine invariants. According to the possible values taken by them in this family we obtain a total of 18 topologically distinct phase portraits. We show that inside the class of all quadratic systems with the topology of the coefficients, there exists a neighborhood of the family of quadratic systems with a weak focus of third order and which may have graphics but no polycycle in the sense of [15] and no limit cycle, such that any quadratic system in this neighborhood has at most four limit cycles.