Complex Uniform Convexity and Riesz Measures

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Abstract. The norm on a Banach space gives rise to a subharmonic function on the complex plane for which the distributional Laplacian gives a Riesz measure. This measure is calculated explicitly here for Lebesgue $L^p$ spaces and the von Neumann-Schatten trace ideals. Banach spaces that are $q$-uniformly PL-convex in the sense of Davis, Garling and Tomczak-Jaegermann are characterized in terms of the mass distribution of this measure. This gives a new proof that the trace ideals $\mathcal{C}^p$ are 2-uniformly PL-convex for $1 \leq p \leq 2$. 

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