A Central Limit Theorem and Law of the Iterated Logarithm for a Random Field with Exponential Decay of Correlations

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Abstract. In [6], Walter Philipp wrote that “…the law of the iterated logarithm holds for any process for which the Borel-Cantelli Lemma, the central limit theorem with a reasonably good remainder and a certain maximal inequality are valid.” Many authors [1], [2], [4], [5], [9] have followed this plan in proving the law of the iterated logarithm for sequences (or fields) of dependent random variables.

We carry on this tradition by proving the law of the iterated logarithm for a random field whose correlations satisfy an exponential decay condition like the one obtained by Spohn [8] for certain Gibbs measures. These do not fall into the $\phi$-mixing or strong mixing cases established in the literature, but are needed for our investigations [7] into diffusions on configuration space.

The proofs are all obtained by patching together standard results from [5], [9] while keeping a careful eye on the correlations.