Admissible Majorants for Model Subspaces of $H^2$, Part I: Slow Winding of the Generating Inner Function

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Abstract. A model subspace $K_{\Theta}$ of the Hardy space $H^2 = H^2(\mathbb{C}_+)$ for the upper half plane $\mathbb{C}_+$ is $H^2(\mathbb{C}_+) \ominus \Theta H^2(\mathbb{C}_+)$ where $\Theta$ is an inner function in $\mathbb{C}_+$. A function $\omega : \mathbb{R} \rightarrow [0, \infty)$ is called an admissible majorant for $K_{\Theta}$ if there exists an $f \in K_{\Theta}$, $f \not\equiv 0$, $|f(x)| \leq \omega(x)$ almost everywhere on $\mathbb{R}$. For some (mainly meromorphic) $\Theta$'s some parts of $\text{Adm}_\Theta$ (the set of all admissible majorants for $K_{\Theta}$) are explicitly described. These descriptions depend on the rate of growth of $\arg \Theta$ along $\mathbb{R}$. This paper is about slowly growing arguments (slower than $x$). Our results exhibit the dependence of $\text{Adm}_B$ on the geometry of the zeros of the Blaschke product $B$. A complete description of $\text{Adm}_B$ is obtained for $B$'s with purely imaginary ("vertical") zeros. We show that in this case a unique minimal admissible majorant exists.