Generalized Factorization in Hardy Spaces and the Commutant of Toeplitz Operators

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Abstract. Every classical inner function \( \varphi \) in the unit disk gives rise to a certain factorization of functions in Hardy spaces. This factorization, which we call the generalized Riesz factorization, coincides with the classical Riesz factorization when \( \varphi(z) = z \). In this paper we prove several results about the generalized Riesz factorization, and we apply this factorization theory to obtain a new description of the commutant of analytic Toeplitz operators with inner symbols on a Hardy space. We also discuss several related issues in the context of the Bergman space.