Some Convexity Features Associated with Unitary Orbits

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Abstract. Let \( \mathcal{H}_n \) be the real linear space of \( n \times n \) complex Hermitian matrices. The unitary (similarity) orbit \( \mathcal{U}(C) \) of \( C \in \mathcal{H}_n \) is the collection of all matrices unitarily similar to \( C \). We characterize those \( C \in \mathcal{H}_n \) such that every matrix in the convex hull of \( \mathcal{U}(C) \) can be written as the average of two matrices in \( \mathcal{U}(C) \). The result is used to study spectral properties of submatrices of matrices in \( \mathcal{U}(C) \), the convexity of images of \( \mathcal{U}(C) \) under linear transformations, and some related questions concerning the joint C-numerical range of Hermitian matrices. Analogous results on real symmetric matrices are also discussed.

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