Octahedral Galois Representations Arising From \(\mathbb{Q}\)-Curves of Degree 2

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Abstract. Generically, one can attach to a \(\mathbb{Q}\)-curve \(C\) octahedral representations \(\rho: \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \rightarrow \text{GL}_2(\overline{\mathbb{F}}_3)\) coming from the Galois action on the 3-torsion of those abelian varieties of \(\text{GL}_2\)-type whose building block is \(C\). When \(C\) is defined over a quadratic field and has an isogeny of degree 2 to its Galois conjugate, there exist such representations \(\rho\) having image into \(\text{GL}_2(\mathbb{F}_9)\). Going the other way, we can ask which mod 3 octahedral representations \(\rho\) of \(\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})\) arise from \(\mathbb{Q}\)-curves in the above sense. We characterize those arising from quadratic \(\mathbb{Q}\)-curves of degree 2. The approach makes use of Galois embedding techniques in \(\text{GL}_2(\mathbb{F}_9)\), and the characterization can be given in terms of a quartic polynomial defining the \(S_4\)-extension of \(\mathbb{Q}\) corresponding to the projective representation \(\bar{\rho}\).