On the Injectivity of $C^1$ Maps of the Real Plane

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Abstract. Let $X: \mathbb{R}^2 \to \mathbb{R}^2$ be a $C^1$ map. Denote by $\text{Spec}(X)$ the set of (complex) eigenvalues of $DX_p$ when $p$ varies in $\mathbb{R}^2$. If there exists $\epsilon > 0$ such that $\text{Spec}(X) \cap (-\epsilon, \epsilon) = \emptyset$, then $X$ is injective. Some applications of this result to the real Keller Jacobian conjecture are discussed.