Convergence Factors and Compactness in Weighted Convolution Algebras

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Abstract. We study convergence in weighted convolution algebras $L^1(\omega)$ on $R^+$, with the weights chosen such that the corresponding weighted space $M(\omega)$ of measures is also a Banach algebra and is the dual space of a natural related space of continuous functions. We determine convergence factor $\eta$ for which weak$^*$-convergence of $\{\lambda_n\}$ to $\lambda$ in $M(\omega)$ implies norm convergence of $\lambda_n * f$ to $\lambda * f$ in $L^1(\omega\eta)$. We find necessary and sufficient conditions which depend on $\omega$ and $f$ and also find necessary and sufficient conditions for $\eta$ to be a convergence factor for all $L^1(\omega)$ and all $f$ in $L^1(\omega)$. We also give some applications to the structure of weighted convolution algebras. As a preliminary result we observe that $\eta$ is a convergence factor for $\omega$ and $f$ if and only if convolution by $f$ is a compact operator from $M(\omega)$ (or $L^1(\omega)$) to $L^1(\omega\eta)$. 

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