Monotone Paths on Zonotopes and Oriented Matroids

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Abstract. Monotone paths on zonotopes and the natural generalization to maximal chains in the poset of topes of an oriented matroid or arrangement of pseudo-hyperplanes are studied with respect to a kind of local move, called polygon move or flip. It is proved that any monotone path on a $d$-dimensional zonotope with $n$ generators admits at least $\lceil \frac{2n}{n-d+2} \rceil - 1$ flips for all $n \geq d+2 \geq 4$ and that for any fixed value of $n-d$, this lower bound is sharp for infinitely many values of $n$. In particular, monotone paths on zonotopes which admit only three flips are constructed in each dimension $d \geq 3$. Furthermore, the previously known 2-connectivity of the graph of monotone paths on a polytope is extended to the 2-connectivity of the graph of maximal chains of topes of an oriented matroid.

An application in the context of Coxeter groups of a result known to be valid for monotone paths on simple zonotopes is included.