

Inequivalent Transitive Factorizations into Transpositions

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Abstract. The question of counting minimal factorizations of permutations into transpositions that act transitively on a set has been studied extensively in the geometrical setting of ramified coverings of the sphere and in the algebraic setting of symmetric functions.

It is natural, however, from a combinatorial point of view to ask how such results are affected by counting up to equivalence of factorizations, where two factorizations are equivalent if they differ only by the interchange of adjacent factors that commute. We obtain an explicit and elegant result for the number of such factorizations of permutations with precisely two factors. The approach used is a combinatorial one that rests on two constructions.

We believe that this approach, and the combinatorial primitives that have been developed for the “cut and join” analysis, will also assist with the general case.

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