K-Theory of Non-Commutative Spheres Arising from the Fourier Automorphism

Samuel G. Walters

Abstract. For a dense $G_δ$ set of real parameters $θ$ in $[0, 1]$ (containing the rationals) it is shown that the group $K_0(\mathcal{A}_θ \rtimes_σ \mathbb{Z}_4)$ is isomorphic to $\mathbb{Z}_9$, where $\mathcal{A}_θ$ is the rotation C*-algebra generated by unitaries $U, V$ satisfying $VU = e^{2\pi i θ} UV$ and $σ$ is the Fourier automorphism of $\mathcal{A}_θ$ defined by $σ(U) = V$, $σ(V) = U^{-1}$. More precisely, an explicit basis for $K_0$ consisting of nine canonical modules is given. (A slight generalization of this result is also obtained for certain separable continuous fields of unital C*-algebras over $[0, 1]$.) The Connes Chern character $ch: K_0(\mathcal{A}_θ \rtimes_σ \mathbb{Z}_4) \to H^{ev}(\mathcal{A}_θ \rtimes_σ \mathbb{Z}_4)$ is shown to be injective for a dense $G_δ$ set of parameters $θ$. The main computational tool in this paper is a group homomorphism $T: K_0(\mathcal{A}_θ \rtimes_σ \mathbb{Z}_4) \to \mathbb{R}^8 \times \mathbb{Z}$ obtained from the Connes Chern character by restricting the functionals in its codomain to a certain nine-dimensional subspace of $H^{ev}(\mathcal{A}_θ \rtimes_σ \mathbb{Z}_4)$. The range of $T$ is fully determined for each $θ$. (We conjecture that this subspace is all of $H^{ev}$.)

Received by the editors November 6, 1999; revised June 14, 2000.
Research partly supported by NSERC grant OGP0169928.
AMS subject classification: 46L80, 46L40, 19K14.
Keywords: C*-algebras, K-theory, automorphisms, rotation algebras, unbounded traces, Chern characters.