Building a Stationary Stochastic Process
From a Finite-Dimensional Marginal

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Abstract. If \(\mathcal{A}\) is a finite alphabet, \(U \subset \mathbb{Z}^D\), and \(\mu_U\) is a probability measure on \(\mathcal{A}^U\) that “looks like” the marginal projection of a stationary stochastic process on \(\mathcal{A}^{\mathbb{Z}^D}\), then can we “extend” \(\mu_U\) to such a process? Under what conditions can we make this extension ergodic, (quasi)periodic, or (weakly) mixing? After surveying classical work on this problem when \(D = 1\), we provide some sufficient conditions and some necessary conditions for \(\mu_U\) to be extendible for \(D > 1\), and show that, in general, the problem is not formally decidable.