Group Actions and Codes

V. Puppe

Abstract. A $\mathbb{Z}_2$-action with "maximal number of isolated fixed points" (i.e., with only isolated fixed points such that $\dim_1(\oplus_i H^i(M; k)) = |M^{\mathbb{Z}_2}|$, $k = \mathbb{F}_2$) on a 3-dimensional, closed manifold determines a binary self-dual code of length $|M^{\mathbb{Z}_2}|$. In turn this code determines the cohomology algebra $H^*(M; k)$ and the equivariant cohomology $H^*_\mathbb{Z}_2(M; k)$. Hence, from results on binary self-dual codes one gets information about the cohomology type of 3-manifolds which admit involutions with maximal number of isolated fixed points. In particular, "most" cohomology types of closed 3-manifolds do not admit such involutions. Generalizations of the above result are possible in several directions, e.g., one gets that "most" cohomology types (over $\mathbb{F}_2$) of closed 3-manifolds do not admit a non-trivial involution.