On the Steinberg Map and Steinberg Cross-Section for a Symmetrizable Indefinite Kac-Moody Group

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Abstract. Let $G$ be a symmetrizable indefinite Kac-Moody group over $\mathbb{C}$. Let $\text{Tr}_{\Lambda_1}, \ldots, \text{Tr}_{\Lambda_{2n-l}}$ be the characters of the fundamental irreducible representations of $G$, defined as convergent series on a certain part $G^{\text{tr-alg}} \subseteq G$. Following Steinberg in the classical case and Brüchert in the affine case, we define the Steinberg map $\chi := (\text{Tr}_{\Lambda_1}, \ldots, \text{Tr}_{\Lambda_{2n-l}})$ as well as the Steinberg cross section $C$, together with a natural parametrisation $\omega: \mathbb{C}^n \times (\mathbb{C}^\times)^{n-l} \to C$. We investigate the local behaviour of $\chi$ on $C$ near $\omega\left((0, \ldots, 0) \times (1, \ldots, 1)\right)$, and we show that there exists a neighborhood of $(0, \ldots, 0) \times (1, \ldots, 1)$, on which $\chi \circ \omega$ is a regular analytical map, satisfying a certain functional identity. This identity has its origin in an action of the center of $G$ on $C$. 

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