Classification of Simple Tracially AF $C^*$-Algebras

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Abstract. We prove that pre-classifiable (see 3.1) simple nuclear tracially AF $C^*$-algebras (TAF) are classified by their $K$-theory. As a consequence all simple, locally AH and TAF $C^*$-algebras are in fact AH algebras (it is known that there are locally AH algebras that are not AH). We also prove the following Rationalization Theorem. Let $A$ and $B$ be two unital separable nuclear simple TAF $C^*$-algebras with unique normalized traces satisfying the Universal Coefficient Theorem. If $A$ and $B$ have the same (ordered and scaled) $K$-theory and $K_0(A)_+$ is locally finitely generated, then $A \otimes \mathbb{Q} \cong B \otimes \mathbb{Q}$, where $\mathbb{Q}$ is the UHF-algebra with the rational $K_0$-theory. Classification results (with restriction on $K_0$-theory) for the above $C^*$-algebras are also obtained. For example, we show that, if $A$ and $B$ are unital nuclear separable simple TAF $C^*$-algebras with the unique normalized trace satisfying the UCT and with $K_1(A) = K_1(B)$, and $A$ and $B$ have the same rational (scaled ordered) $K_0$, then $A \cong B$. Similar results are also obtained for some cases in which $K_0$ is non-divisible such as $K_0(A) = \mathbb{Z}[1/2]$. 