Well Ramified Extensions of Complete Discrete Valuation Fields with Applications to the Kato Conductor

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Abstract. We study extensions $L/K$ of complete discrete valuation fields $K$ with residue field $\mathfrak{K}$ of characteristic $p > 0$, which we do not assume to be perfect. Our work concerns ramification theory for such extensions, in particular we show that all classical properties which are true under the hypothesis "the residue field extension $L/\mathfrak{K}$ is separable" are still valid under the more general hypothesis that the valuation ring extension is monogenic. We also show that conversely, if classical ramification properties hold true for an extension $L/K$, then the extension of valuation rings is monogenic. These are the "well ramified" extensions. We show that there are only three possible types of well ramified extensions and we give examples. In the last part of the paper we consider, for the three types, Kato's generalization of the conductor, which we show how to bound in certain cases.